



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 55

Issues in Futures Hedging

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WHY FUTURES HEDGE IS NOT A PERFECT HEDGE?

- Marking to market.
- **Basis risk due to:**
- Different underlying asset.
- Non-identical maturity.
- Lot size issue.



CHOICE OF UNDERLYING ASSET

- Choose
- Either the same asset as the asset being hedged or
- An asset whose futures price has a high positive / negative correlation with the spot price of the asset to be hedged.



- If the underlying asset is positively correlated with the exposed asset, the hedge must be opposite to the exposure i.e. long exposure to be covered by short futures hedge.
- If the underlying asset is negatively correlated with the exposed asset, the hedge position must be the same as the exposure i.e. long exposure to be covered by long futures hedge.
- The basic philosophy is that the effect of hedged risk on the exposure should be annulled by the effect of the risk on the hedging asset.



CHOICE OF MATURITY MONTH OF THE FUTURES CONTRACT

- We illustrate this exercise by an example:
- Assume that today is February 28. You have just contracted a 3-month USD payable. It will mature for payment on June 1.
- There is no USD futures contract maturing on June 1. Traded USD futures mature on the third Wednesdays of June, September, December & March.
- You decide to hedge your payable by **BUYING** USD futures.



- Suppose that on Feb 28, the spot USD rate is Rs 70 per USD and the futures rates are:
- | Maturity | Rate | Basis |
|-----------|-------|-------|
| June | 70.25 | -0.25 |
| September | 70.75 | -0.75 |
| December | 72.00 | -2.00 |
- The problem is to decide which maturity's futures should you buy.



HOW WILL THIS HEDGE OPERATE?

- Since you have a USD payable on June 1, you will lose if the USD appreciates, so you will hedge against a price rise in USD i.e. the hedge must yield a profit if USD appreciates.
- Hence, you will **buy USD futures** on Feb 28 because you will make a profit if USD appreciates on this futures position.
- This profit will compensate you for the loss on your primary position if USD appreciates.



- **On the delivery date, you will make purchases of the USD in the spot market. At the same time, you will lift the hedge by closing out the futures position.**
- **Any loss on the spot position would be compensated by gains in the futures position and vice versa.**

THE QUANTITATIVE ANALYSIS

- Price change on the spot position $\Delta S = S_T - S_0$.
- Price change on the futures position $\Delta F = F_T - F_0$
- Hence, for an effective hedge, we must ideally have: $\Delta F - \Delta S > 0$ or $F_T - F_0 > S_T - S_0$ or $S_0 - F_0 > S_T - F_T$
- i.e. **BASIS AT $t=0$ (Feb 28) > BASIS AT $t=T$ (June 1)**



- We are given that on Feb 28, the spot USD rate is Rs 70 per USD and the futures rates are:

• Maturity	Rate	Basis
• June	70.25	-0.25
• September	70.75	-0.75
• December	72.00	-2.00



- For hedge effectiveness, we want
- $\text{BASIS AT } t=0 \text{ (Feb 28)} > \text{BASIS AT } t=T \text{ (June 1)}$
- Since, the basis are all negative at $t=0$ (Feb 28), it is reasonable to assume that they will continue to be negative at $t=T$ (June 1).
- Thus, we want, $\text{MAGNITUDE OF BASIS AT } t=0 \text{ (Feb 28)} < \text{MAGNITUDE OF BASIS AT } t=T \text{ (June 1)}$.

- **Now, as on June 1, the June futures contract would be very close to maturity. Hence, its price would be very nearly converging to the spot price, so that the basis of the June futures would be very small in magnitude and hence, undesirable from the hedge effectiveness point of view.**



- It would be more appropriate to go for a distant maturity contract e.g. the Sept/Dec futures rather than the June futures in this problem.
- However, one needs to keep in mind that distant maturity futures tend to be less liquid and also have larger frictional losses e.g. bid-ask spreads etc.

	SHORT HEDGE	LONG HEDGE
POSITIVE BASIS	DISTANT	NEAR
NEGATIVE BASIS	NEAR	DISTANT



NUMBER OF CONTRACTS

Let $\tilde{Z} = \alpha\tilde{X} + \beta\tilde{Y}$. then

$$\begin{aligned}\sigma_Z^2 &= \sigma_{\alpha X + \beta Y}^2 = E[\alpha X + \beta Y - E(\alpha X + \beta Y)]^2 \\ &= E[\alpha X + \beta Y - \alpha E(X) - \beta E(Y)]^2 = E\{\alpha[X - E(X)] + \beta[Y - E(Y)]\}^2 \\ &= E\{\alpha[X - E(X)]\}^2 + E\{\beta[Y - E(Y)]\}^2 \\ &\quad + 2E\{\alpha[X - E(X)]\beta[Y - E(Y)]\}\end{aligned}$$



$$\begin{aligned}
&= \alpha^2 \mathbf{E} [\mathbf{X} - \mathbf{E}(\mathbf{X})]^2 + \beta^2 \mathbf{E} [\mathbf{Y} - \mathbf{E}(\mathbf{Y})]^2 \\
&+ 2\alpha\beta \mathbf{E} \{ [\mathbf{X} - \mathbf{E}(\mathbf{X})][\mathbf{Y} - \mathbf{E}(\mathbf{Y})] \} \\
&= \alpha^2 \sigma_X^2 + \beta^2 \sigma_Y^2 + 2\alpha\beta \sigma_{XY} \\
&= \alpha^2 \sigma_X^2 + \beta^2 \sigma_Y^2 + 2\alpha\beta \rho_{XY} \sigma_X \sigma_Y
\end{aligned}$$

$$\Delta I = Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F}$$

$$\sigma_{\Delta I}^2 = Q_S^2 \sigma_{\Delta S}^2 + Q_F^2 \sigma_{\Delta F}^2 - 2\rho Q_S Q_F \sigma_{\Delta S} \sigma_{\Delta F}$$

For minima $\frac{d\sigma_{\Delta I}^2}{dQ_F} = 0 = 2Q_F \sigma_{\Delta F}^2 - 2\rho Q_S \sigma_{\Delta S} \sigma_{\Delta F}$

$$Q_F = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} Q_S = \beta_{\Delta S, \Delta F} Q_S = h Q_S \quad (1)$$

By definition: $\frac{Q_F}{Q_S} = h = \text{hedge ratio}.$

Thus, in the special case of minimum variance hedge

$$Q_F = hQ_S = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} Q_S \text{ whence } \frac{Q_F}{Q_S} = h = \text{hedge ratio} = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = \beta_{\Delta S, \Delta F}$$

$$\text{Also } \frac{d^2 \sigma^2}{dQ_F^2} \Delta I = 2\sigma_{\Delta F}^2 > 0 \text{ confirming minimality of variance}$$

IMPORTANT

- The regression is with changes in prices and not prices themselves.
- The futures price changes is the independent variable and the spot price changes is the dependent variable.
- We have not accounted for the integrality of the no of contracts.

